Equity Valuation: to Bayes or not to Bayes?

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Abstract

This paper discusses the benefits and drawbacks of adopting Bayesian inference for equity valuation and derives a decision-focused valuation model for researchers and investors. Despite the inevitably subjectivist (i.e. Bayesian) nature of equity valuation, almost all research contributions on the topic follow a frequentist approach. In this paper, I perform a methodological discussion on equity valuation and provide a starting point for a more investor-oriented equity valuation in both theory and practice. I argue that valuation theory alone supplies enough information to generate suitable, normally distributed weakly informative priors for long-term speculative variables in terminal values. When combined with maximum-likelihood estimates using historical fundamentals and/or short-term estimates, these simple-to-implement priors should lead to a more reliable estimation of intrinsic value. Using the specified prior and likelihood, I propose a Bayesian algorithm for the simulation of the posterior distribution of intrinsic value.

Contents

1. Introduction

Investors seeking lucrative equities face decision problems under uncertainty, and capital-market based accounting and finance research (CMAFR) has achieved significant advances related to those decision problems over the last century. To name just a few much-cited examples, CMAFR includes research concerning…

- the efficiency of capital markets to specific information (Efficient Market Hypothesis (EMH)), based on Fama (1970),
- asset pricing models anchoring on expected returns, e.g. Sharpe (1964) and
- discounted payoff valuation models such as Gordon (1959), Ohlson (1995) or Ohlson & Juettner-Nauroth (2005).

They were followed by large streams of empirical research that examine the practical validity of these theories and models:

- studies that found evidence in favour of the EMH (risk-based explanations for anomalies, in line with "classic" finance, e.g. Fama & French (1992)) or the opposite (mispricing-based explanations for anomalies, in line with behavioural finance, e.g. Huefner & Rueenaufer (2021)),
- several different factor models for assets pricing based on the Capital Asset Pricing Model (CAPM) such as Carhart (1997) or
- studies that employ discounted payoff models to value equities on larger scales, e.g. Penman & Sougiannis (1998), Frankel & Lee (1998), Francis et al. (2000) or Jorgensen et al. (2011).

At least some of these theoretical and empirical advances results have undoubtedly affected capital markets around the globe.[1] More recently however, a stream of research has started raising severe concerns about the dominant "frequentist" methodology in empirical CMAFR because it anchors on p-values and large sample sizes in null-hypothesis significance tests (NHST), like Lindsay (1995), Basu (2015), Kim & Ji (2015), Dyckman (2016), Kim (2018), Kim et al. (2018), Ohlson (2020), Johannesson et al. (2020), Johnstone (2021) and Michaelides (2021). The outcome of a concentration on these methods is an increased occurrence of type I errors – or "false positives" – of tested hypotheses. This is problematic since researchers tend to solely focus on statistical significance without considering economic significance. In consequence, many of the empirical insights in the field become questionable:

> "*… most of the current literature will, in due course, be dismissed as at best dubious.*" – Ohlson (2020).

As potential remedies, the pertinent literature suggests different alternatives. Some contributions extend the current reporting strategy of statistics by confidence intervals to include effect sizes (e.g. Dyckman (2016)) or apply incremental analyses to examine the effective explanatory power of main variables of interest (e.g. Johannesson et al. (2020)). Interestingly, several of these studies also mention (and sometimes briefly discuss) the possibility of applying Bayesian statistics as an alternative to frequentist statistics. Advocates of Bayesian thinking have long criticised the limited ability of frequentist methods to resolve decision problems under uncertainty, e.g. Winkler (1973), Cousins (1995), Goldstein (2006) or Johnstone (2018).

[1] Pinto et al. (2019) have recently conducted a survey that questioned analysts on their methodology, and they do employ valuation methods that stem from these research advances, like different factor models (e.g. the Capital Asset Pricing Model (CAPM) or discounted payoff models like the Discounted Cash Flow Model (DCF) or the Residual Income Valuation Model (RIM).

Indeed, equity valuation in both research and practice is a quite prominent decision-problem under a large amount of uncertainty; particularly terminal values in discounted payoff equity valuation models require users to make very impactful assumptions on the long-term behaviour of difficult-to-predict input variables. It might therefore be interesting to examine whether Bayesian methodology can be used to improve practical equity valuation. The emphasis of this paper lies on practical equity valuation and thus assumes the perspective of an investor, but many of the points made in the discussion apply to empirical equity valuation research as well, given that it ultimately involves similar decision problems on a larger scale. In order to determine whether Bayesian inference can be helpful for investors, three research questions arise:

(1) How did the dominance of frequentist methods in the field of equity valuation come to exist and why does it persist until today?

Finding an answer to this question requires a closer look at the history of statistics and methodology in capital market research and investment practice. I will only give a brief outline here because Johnstone (2018) provides an excellent and thorough timeline of the history of the two statistical camps in the context of accounting and capital markets. Following Sloan (2019), the constant shift towards rule-based, quantitative strategies started with the introduction of Markowitz' portfolio-theory, CAPM and the EMH in the 1960s and 1970s. Since computational power increased more and more over the following decades and market-wide databases were developed (e.g. Compustat, CRSP or I/B/E/S), it became much easier for investors to perform simplified fundamental and technical analyses that exploited empirically-documented anomalies or focused on passive investment styles due to the assumptions of EMH. The rise of these strategies was the fall of comprehensive fundamental analyses as described by Graham & Dodd (1934), so much so that textbooks in finance simply replaced the sections covering fundamental analyses by technical analysis tools. In terms of statistical methodology, frequentist methods were adequate for these simple-to-compute applications because they relied heavily on larger amounts of data, both in cross-section and time-series. As such, there was no necessity to put the methodology in question and search for an alternative, so that Bayesian analysis was not taught much. One of the main issues of Bayesian analysis is that it is often more computationally intensive, so it simply was not executable at the time. Nowadays however, that is not an obstacle anymore, which is the reason why many other domains (e.g. marketing or economics) have explored Bayesian methodology more and more.[2] In CMAFR, despite the issues of frequentist methods mentioned before, Bayesian analysis remains relatively scarce.[3]

(2) Do frequentist or Bayesian viewpoints suit equity valuation in practice? Is there a thorough methodological discussion that answers that question?

Since the general debate between the two methodologies in research has been going on for a century across several domains, there are too many relevant papers and books to cite them all here.[4] In the context of stock price forecasting or portfolio selection, other contributions do provide arguments that promote Bayesian superiority:

i. The capability of Bayesian inference to update probability distributions of input parameters on portfolio levels, e.g. Winkler (1973), Winkler & Barry (1975) and Cornell (2020),

[2] Poirier (2006) and Basturk et al. (2014) show an increasing tendency of usage of the terms "Bayes" or "Bayesian" and increased devotion to Bayesian concepts in statistics and economics over the time period of 1968 to 2014.
[3] Beck et al. (2012) examine the use of Bayesian analysis in finance and come to similar conclusions.
[4] Discussions in other domains would be, among many others, Midway (2019) – Ecology, Rossi & Allenby (2003) – Marketing, Ambaum (2012) – Meteorology or Cousins (1995) – Physics.

ii. a better representation of model uncertainty in forecasting in Bayesian statistics compared to frequentist practices, e.g. Avramov (2002), Bird & Gerlach (2003), Yee (2008a), Avramov & Zhou (2010) and Yee (2010) and

iii. more accurate stock price forecasting when using Bayesian modeling, Ying et al. (2005), Higgins et al. (2006), Yee (2010) and Zuo & Kita (2012).

However, they do not provide a thorough methodological discussion of the two statistical camps in the context of applied equity valuation via discounted payoff models.[5] As a result, closing that research gap is the first contribution of this paper in Section 3. Before I do that, I provide a theoretical fundament for the discussion in Section 2, which includes the key characteristics of fundamental investors and an outline of an ideal-typical process of equity valuation based on fundamental data.

(3) Is there an equity valuation model that supports decision-making on capital markets? If not, what could be done to improve the situation?

A comprehensive fundamental analysis typically leads to an investment decision made by the investor, preceded by the choice of an equity valuation model (or multiple for that matter). As such, the investor faces the problem of choosing a model that best supports the decision-making process. To my knowledge, there is no proposal for a stand-alone valuation model that a) uses discounted payoffs and b) suitably supports the decision-making of investors through its conceptualisation by accounting for the large amount of uncertainty within the process. Ohlson (2005), Penman (2005) and Huefner & Rueenaufer (2020) provide discussions of existing accounting-based valuation models, but without a clear focus on the models' ability to support decisions under uncertainty. Chen & Schipper (2016) argue that existing valuation models – like the prominent examples derived in Ohlson (1995) or Ohlson & Juettner-Nauroth (2005) – are silent on the information that determines the expected payoffs and do not include any particular foundation for decisions (e.g. an implementation of Bayes' rule or guidance on designs of null hypothesis testing). Winkler (1973), Winkler & Barry (1975) and Avramov & Zhou (2010) provide Bayesian models for portfolio selection. Their models, though, are more of an overarching framework for portfolio management than a useful link between a particular set of information and firm value.

Therefore, I argue that the existing models and investment approaches are no sufficient baseline for the fundamental investor, particularly when it comes to choices made in terminal values. My second contribution in Section 4 is to derive a Bayesian valuation model that provides investors with a decision-focused framework and links intrinsic values and expected returns to fundamental accounting information. Finally, Section 5 demonstrates the Bayesian simulation algorithm applied to the previously derived model and showcases its utility through a practical example.

[5] Cornell (2020) applies a form of Bayes' rule that only uses historical stock price data and applies the CAPM to infer an expected return for comparison. That is not reflective of the emphasis of analysis of fundamental investors. Cochrane (2011) also criticises CAPM-centred valuation: *"When did our field stop being asset pricing and became asset expected returning?"*. Bird & Gerlach (2003) and Yee (2008a) perform Bayesian approaches to apply simplified fundamental analyses through model averaging and Bayesian triangulation. They also provide some arguments for why the Bayesian modelling is advantageous to other "frequentist" methods. While their models could be useful for stock screening, they trivialise the procedure of fundamental analysis to a solely quantitative approach when used as stand-alone models.

2. *Theoretical Fundament*

2.1. Key Assumptions and Outline of Fundamental Analysis

As a baseline for the conceptual discussion and subsequent model derivation, I use the following set of assumptions:

1. The future is uncertain, so market participants (e.g. analysts, investors, firms etc.) act under uncertainty. In turn, while the families of the true probability distributions of prospective parameters are assumed certain, the exact distribution within the respective family is uncertain.
2. Fundamental investors are risk-averse and strive to behave rationally; they aim to maximise their personal utility function.[6]
3. Investors do not know what information is used by other investors and cannot assess the decision-usefulness of information with absolute certainty, such that expectations on the future are heterogeneous.[7]
4. Under information asymmetry, uncertainty and heterogenous expectations, capital markets are not always semi-strongly efficient, so that market participants do not always incorporate all publicly available information rationally and efficiently into prices.
5. Fundamental investors anchor their estimate of intrinsic value on the expected amount, timing, and uncertainty of future expected payoffs. These payoffs affect intrinsic value through a valuation model that discounts them to the present date.
6. Fundamental investors expect market prices to converge to the estimate of intrinsic value, starting at the investment date $t = 0$ and ending at some future date $t = 1$.

Under those circumstances, fundamental investors face a series of decision problems under uncertainty when trying to assess firm value. Under uncertainty, the probability distribution of intrinsic value is uncertain (and remains uncertain), because intrinsic value is unobservable. To provide a clearer idea of the series of decision problems that investors face and the types of uncertainty within them, I describe an ideal-typical process of fundamental analysis as follows – similar to textbooks like Penman (2013), pp. 85 & 86:

Step 1: Stock Screening

First and foremost, investors need to identify stocks that appear as lucrative candidates for the comprehensive fundamental analysis. In order to do so, a calibrated, simplified analysis of selected fundamentals and key value drivers (through scoring systems and multiples for example) can be used to screen stocks in a quantitative manner.[8] The decision problems here typically involve the selection of key metrics and ratios for the classification of firms and the selection of a small number of stocks for the comprehensive analysis of fundamental information.

Step 2: Analysis of Fundamental Information

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[6] This is in line with the expected utility theory developed by Savage (1954), which traces back to the 17th century, where Blaise Pascal and Pierre de Fermat developed foundations of decision-making under uncertainty. Due to Jensen's inequality, investors also prefer investments with lower risk-reward trade-offs.

[7] Johnstone (2015) defines information uncertainty as "*the risk of a misleading signal*". Other definitions of information uncertainty are rather vague. For example, Jiang et al. (2005) define it as "*the precision with which firm value can be assessed by knowledgeable investors at reasonable cost*". This definition subsumes several other sources of uncertainty that can also be regarded separately.

[8] Huefner & Rueenaufer (2021) provide an approach for stock screening for investors that anchors on errors-in-expectations and combines quality and cheapness. Also see Piotroski (2000), Mohanram (2005), Piotroski & So (2012), Asness et al. (2019) and Li & Mohanram (2019) for similar techniques.

Once a few lucrative candidates have been chosen, investors take a deeper dive into the business. This second step is multi-faceted and represents the backbone of fundamental analysis. Here is where investors try to reduce information uncertainty by processing decision-useful information for the ensuing forecasting process. Doing that requires a careful examination of the business strategy, accounting, and financial performance of the firm, as e.g. Sloan (2019) argues. Similar to stock screening, the main problem for investors on this step is that they need to decide which information to use for the inference, but cannot assess with certainty the quality of the information they receive and use.

Step 3: Choice and Conceptualisation of a Valuation Model

In the process of forecasting, fundamental investors have a selection of valuation models they can choose from (as the "architecture" of fundamental analysis). This process also includes a conceptualisation of the components inside the model: anchoring value, short-term expectations, and long-term speculation. Investors therefore also face a problem of model uncertainty, as investors do not know a-priori which model and conceptualisation best support the predictive task. Theoretically, all existing discounted payoff valuation models equate to the present value of expected dividends (PVED) and are thus mathematically equivalent. However, as demonstrated by Penman (1998) and Courteau et al. (2001), they can still yield different results if the terminal values in them are not used in a consistent way. Also see Penman & Sougiannis (1998) for an empirical comparison and Lundholm & O'Keefe (2001) and Penman (2001) and references therein for discussions.

Step 4: Parameter Operationalisation

Afterwards, fundamental information is condensed into beliefs on the amount, timing, and uncertainty of future expected payoffs, both in the short-term and the long-term.[9] Together with the anchoring value, these beliefs then flow into the valuation model(s) of choice and result in an estimate (of the probability distribution) of intrinsic value. Forecasting is probably the most difficult task within fundamental analysis, and it is hard to provide universally applicable guidance on the process. Textbooks on fundamental analysis typically include an extensive chapter on the subject, but need to remain vague in terms of explicit advice for individual inferences. That is because prospective parameters (e.g. future earnings or dividends) are highly uncertain and change over time, so there is a large amount of parameter uncertainty involved.[10]

Step 5: Decision-making

Inference requires conclusion, and the main conclusion in fundamental analysis is the investment decision. Investors make guesses on the difference between price and intrinsic value; what is the most probable intrinsic value? How certain is the investor that the intrinsic value exceeds (or falls short of) the market price? Any probability assessment requires a quantification of uncertainty, and in this case, also entails a quantification of the beforementioned information uncertainty, model uncertainty, and parameter uncertainty. The Margin of Safety (MoS) introduced by Graham & Dodd (1934) is an (informal) outcome of that.

Step 6: Monitoring

After the investment decision, investors monitor the stock price and incorporate newly arriving information to revise their previous position. Just like on the first two steps, information uncertainty affects this process. But not

[9] Anchoring on Graham & Dodd (1934), Penman (2011) reminds investors to not "*mix what you know with speculation*".

[10] See, for example, the respective chapters in Penman (2013), Easton et al. (2018), Wahlen et al. (2018) or Palepu et al. (2019).

only that: Yee (2008b) highlights that intrinsic value is subject to two additional sources of uncertainty: uncertainty of news and convergence. Since future events and information related to those events are unknown, incoming news between the investment date and the expected date of convergence may significantly change the estimate of intrinsic value. News uncertainty is closely linked to information uncertainty, since investors need to assess the decision-usefulness of newly arriving information before they can determine the impact on the previous estimate of intrinsic value and revise their position accordingly. Convergence uncertainty simply means that investors do not know the pace of the expected convergence with certainty.

2.2. Implications for the Fundamentalist Discussion

Following de Bruin et al. (2018), all choices made by market participants under uncertainty are, in essence, somewhat subjective.[11] Research suggests that the choice of decision-useful information depends on various personal characteristics of the investor, such as wealth, the attitude towards risk and the path of education. These factors also shape the preferences of investors concerning certain investment styles (like value, growth, momentum etc.).[12] The outcome of that are potential differences in intrinsic value assessments across investors, even under rationality:

> "*However, if subjectivity is involved in the calculation of intrinsic value, then two investors can, rationally, disagree about the value of a security.*" – Greene (2019).

It is evident that the choice of valuation model and parameter operationalisation alone have significant effects on the outcome. Pinto et al. (2019) provide a recent survey of professional valuation practice that shows that while there are dominant practices, analysts do use a large variety of methods to value equities. It is thus fair to say that investors make subjective probability assessments under uncertainty when estimating intrinsic value. The goal of the ensuing discussion is to evaluate whether frequentist or Bayesian approaches better support subjective probability assessments under uncertainty. In other words: does frequentist or Bayesian methodology suit equity valuation? To answer that question, I describe and discuss the handling of (the various sources of) uncertainty, the possibility to include information outside of the data (information that can be perceived as subjective) and the interpretation of probability in frequentist and Bayesian thinking. It is worth mentioning that aside from the apparent practical relevance for investors, analysts and academics on equity markets, these topics are prominent in methodological discussions of Bayesian and frequentist approaches across many different domains of research.[13]
Figure 1 shows the outline of the process of fundamental analysis, including the type of analysis, sources of uncertainty, steps on the inference process and the inherent practical decision problems.

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[11] Graham & Dodd (1934) and Williams (1938) state that market prices are based on emotions and popular opinion, also supporting the hypothesis that subjectivity affects market prices.

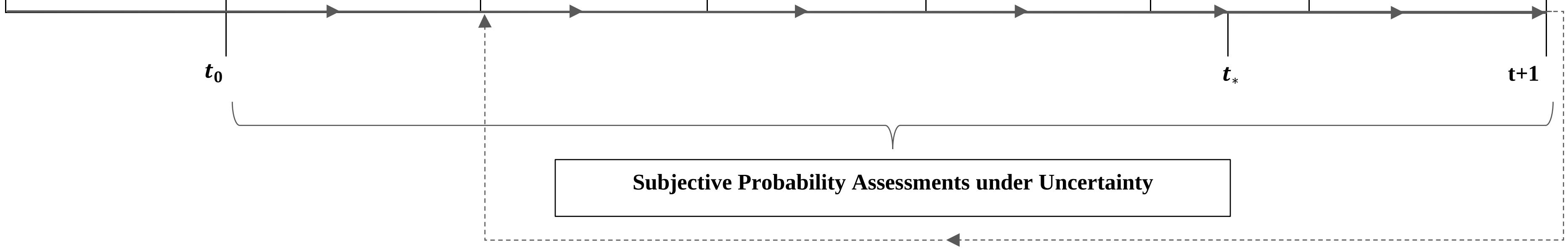
[12] See Barberis & Shlaifer (2003), Kumar (2009a), Kumar (2009b), Cronqvist & Siegel (2014), Cronqvist et al. (2015) and Greene (2019) and references therein for insightful perspectives on the backgrounds and preferences of individual investors.

[13] I counted words related to methodology in over 50 papers and textbooks that include the words "Bayesian" and "frequentist" simultaneously. A full list of references and the word count is available upon request. Note that the word "frequentist" is usually only used by Bayesian statisticians to describe "classical", "orthodox", "objectivist" and "sampling-theoretic" statistics. As such, the word count might include a bias towards Bayesian literature. Nonetheless, the relevance of these topics is evident.

Figure 1:

Outline of Fundamental Analysis

Type of Analysis	Business Analysis		Prospective Analysis		Investment Decision + Revision	
Main Source of Uncertainty	Information Uncertainty		Model Uncertainty	Parameter Uncertainty	News Uncertainty	
Step of Inference	Stock Screening	Processing of Fundamental Information	Modelling and Conceptualisation	Operationalisation of Parameters	Investment Decision	Monitoring
Decision Problem	Selection of Metrics for Simplified Fundamental Analysis and Initial Stock Selection	Selection of Decision-useful Information for Comprehensive Business Analysis	Selection of Valuation Model and Conceptualisation of Building Blocks	Estimation of Predictive Parameters based on Business Analysis	Investment Decision (Buy / Short-Sell / Inaction)	Revision based on new Decision-useful Information (Buy/ Hold / Sell)
Step #	1	2	3	4	5	6



3. *Fundamentalist Discussion of Bayesian and Frequentist Approaches to Equity Valuation*

3.1. The Understanding of Probability

As I mentioned in the introduction, the standard procedure in empirical CMAFR is null hypothesis significance testing (NHST).[14] NHST induces an emphasis on statistical significance over economic significance: even if a slope coefficient in a simple linear regression is different from zero and statistically significant (i.e. has a p-value close to zero), that does not have to be indicative of a somewhat interpretable economic relation between the two parameters of interest. In the context of the understanding of probability, the p-Value is exemplary for frequentist practices: it stands for the probability of observing the data at hand (or more extreme data), given the formulated hypothesis (and model). Formally, this equates to p(D|H), where D represents the data and H the hypothesis.[15] The frequentist idea of probability – found in Neyman (1937) and von Mises (1961) for example – interprets probability as the limit of a relative frequency in repeated trials of an experiment. As such, "true and objective" probability can be discovered by repeating an experiment an infinite number of times (or having "all the data") because then the change in relative frequency converges to zero.

But does this take on probability appeal to a fundamental investor, especially when considering the valuation problem? In practice, decision-making under uncertainty requires the investor to see equities as more than a repeatable experiment. Every firm is unique in that it requires the investor to process a different set of information to make guesses about its intrinsic value (e.g. concerning its business strategy, accounting policies or financial performance). Regarding a firm as "just another fish in the sea" does not align well with the foundations of fundamental analysis. The frequentist interpretation of probability runs into issues when facing small sample sizes and various sources of heterogeneity in the data (as p(D|H) is then unreliable), and both are natural consequences of valuations tailored to each individual firm.

Under subjectivist Bayesian rationale à la de Finetti (1964), probability is interpreted as the quantification of personal beliefs or expectations under a given state of knowledge.[16] The fundament for that understanding is Bayes' theorem:

$$\mathrm{p}(\mathrm{H}|\mathrm{D}) = \frac{\mathrm{p}(\mathrm{D}|\mathrm{H})\mathrm{p}(\mathrm{H})}{\mathrm{p}(\mathrm{D})}. \tag{1}$$

Here, p(H) is the prior probability that includes the knowledge prior to observing the data, p(D|H) is the likelihood that can be understood frequentist interpretation of probability, the marginal probability p(D) and the posterior probability p(H|D) that combines prior, likelihood and the marginal probability. The latter only has a normalising effect on the outcome and is thus often excluded, resulting in the proportional conditional posterior probability:

$$\mathrm{p}(\mathrm{H}|\mathrm{D}) \propto \mathrm{p}(\mathrm{D}|\mathrm{H})\mathrm{p}(\mathrm{H}). \tag{2}$$

Based on the theorem, Bayesian probability generalises frequentist probability since it includes more than p(D|H). At the same time, probabilities can be assigned to different, alternative hypotheses and then these hypotheses can

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[14] Dyckman (2016) examined 460 papers in CMAFR over the timeframe 2011 to 2014 and all of them, without exception, applied NHST.

[15] It would be more exact to regard D as a representation of a test statistic, since data cannot be "extreme" in a statistical sense.

[16] Kaplan (2014) also refers to subjective probability as epistemic probability based on Howson & Urbach (2006).

be evaluated against each other, e.g. through the Bayes factor.[17] Formally, as is visible in Equation (1), it is possible to determine p(H|D) directly, e.g. in form of the probability distribution of intrinsic value given the available information. Since frequentists only focus on the inverse probability p(D|H), they cannot answer the main questions that investors ask when trying to assess intrinsic value: what is the probability of intrinsic value to exceed (or fall short of) the market price? It is thus evident that if investors strive to resolve their practical decision problem under uncertainty, it would be more intuitive to follow the more general Bayesian understanding of probability.[18]

3.2. The Inclusion of Subjective Information

When looking at the discussion between the two "camps" in statistics, one could argue that "*frequentists are objectivists".* After all, it is understandable that valuation practice should remain objective in principle. Indeed, Graham & Dodd (1934) remind investors to anchor their valuation on the "relevant facts". Frequentists let the data speak for itself and only rely on p(D|H). Bayesians on the other hand argue – in the spirit of Lindley & Smith (1972) – that there typically is information available that does not come from the data, but allows for better estimates when combined with the data through a formal representation of prior knowledge via p(H). On fast-moving capital markets, ignoring information that does not come from the data can in fact be dangerous (like ignoring information on accounting fraud that is not yet reflected in the balance sheet).[19] Unfortunately, frequentist statistics do not supply an architecture that enables an inclusion of information outside of the data, because that is seen as too subjective.[20]

Nonetheless, it is true that this subjective, hard-to-quantify kind of information may standardise or skew the posterior if the choice of prior distribution is too informative (e.g. as a result of overconfidence in the face of high uncertainty), especially when there is not a lot of evidence available.[21] In consequence, one of the most dominant points of criticism that advocates of frequentist statistics make is that Bayesian analysis is too subjective. Due to the prospective nature of equity valuation however, it inevitably entails subjective estimates concerning the choice of decision-useful information, the date of convergence, the valuation model and parameters that the output is very sensitive to, e.g. expected long-run growth rates in payoffs. In practice, those will vary from investor to investor:

> *"In the face of uncertainty and heterogeneous beliefs or information asymmetry, estimated value – like beauty – is in the eyes of the beholder."* – Yee (2008a).

So even if one applies a strictly frequentist framework and methodology to equity valuation, it is ultimately necessary to make quite subjective judgements. As such, one could say that frequentists start from more objective positions via p(D|H), but then they end up "diluting" their analysis with subjective assumptions that they cannot avoid anyway. Bayesians start on a more subjective ground through the inclusion of p(H), but with more and more evidence, they move back into a more objective reality as the weight of p(D|H) increases. As Johnstone (2018) argues, the Bayesian viewpoint is that subjectivity is unavoidable and should be treated with transparency, leading

[17] Kim et al. (2018) suggest Bayes factor as an alternative to classical p-Value for significance testing. It is important to acknowledge that the quality of one hypothesis cannot be judged without comparing it to at least one alternative, as Dyckman (2016) states. Basturk et al. (2014) also highlight that without a measure of credibility – e.g. a comparison to other models – rejecting a null hypothesis has no direct meaning.

[18] Under Bayesian rationale, all probability is subjective, just the degree of subjectivity varies.

[19] To quote Penman (2011): "*Ignore information at your peril*".

[20] There are various recurring subjective assumptions in studies that apply large data samples to valuation problems. To name two prominent examples, the choices regarding the exclusion of certain observations (winsorisation) and choice of significance level is always subjective and often more or less arbitrary. See Gassen & Veenman (2021) for a recent discussion on the importance of outliers and winsorisation in accounting research.

[21] See McNeish (2016) for a more extensive review on the importance and problems of prior selection for small samples.

to better modelling and practical application.[22] That does not mean that more subjectivity is always preferable; it implies that it should be included as much as necessary whenever it cannot be avoided. Overall, I see a clear advantage for Bayesian approaches for fundamental analysis in terms of the inclusion of subjective information.

3.3. The Handling of Uncertainty

As I stated before, any rational decision-making process – which is usually the desired conclusion of the fundamental analysis – requires a consideration of the uncertainty:

> "*The field of investment analysis provides an example of a situation in which individuals or corporations make inferences and decisions in the face of uncertainty about future events. The uncertainty concerns future security prices and related variables, and it is necessary to take account of this uncertainty when modelling inferential or decision-making problems relating to investment analysis. Since probability can be thought of as the mathematical language of uncertainty, formal models for decision making under uncertainty require probabilistic inputs.*" – Winkler (1973).

Referring to Section 2, I consider four types of uncertainty that have an impact on the inference: information uncertainty, model uncertainty, parameter uncertainty and news uncertainty.

Information Uncertainty

Information is the main resource in everyday business on capital markets.[23] With more information comes a better understanding of the firm (on average) because both p(H) and p(D|H) become more precise, and with a better understanding of the firm, p(H|D) also becomes more precise.[24] Assuming information asymmetry and uncertainty about the future however, investors cannot assess with certainty the decision-usefulness of information they receive. Information uncertainty therefore hinges on several "qualities" of the sum of decision-useful signals, i.e. the prospective degree or the presence of incentives for opportunistic behaviour of the senders. Either problem may lead to biased information. It is commonly understood that a) the market mainly prices the future and b) that market participants often have incentives to behave opportunistically. In order to demonstrate b), modern agency-theory à la Jensen & Meckling (1976) best illustrates the relation between market participants: investors serve as the principal and other participants such as firms or equity analysts – serve as the agents. The opportunistic agents supply investors with information on the firm, leading to potential conflicts of interest. Thus, it is fair to say that signals sent by firms or analysts may very well be biased and subject to information uncertainty. Accounting is in fact known to be conservative (often for a good reason) and includes discretionary elements. Further, it is well-documented that analyst forecasts have been optimistically biased over the last few decades. [25]

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[22] Goldstein (2006) also emphasises that particularly for the resolution of many practical decision problems, a subjective Bayesian analysis is the only feasible method. Many arguments in his analysis can be transferred to equity valuation as well.

[23] Investors increase their knowledge on the firm during the analysis and then update their expectations on the Fisher (1930) triad (amount, timing and uncertainty) of future payoffs. The triad then determines the posterior distribution of intrinsic value, which allows investors to obtain probabilities of it exceeding (or falling short of) the current market price.

[24] It is possible that more information on a particular firm increases uncertainty, as Beaver (1968) states. The common notion in the field of research is that more or higher-quality information reduces the assessment of risk through the cost of capital – see Botosan (1997), Lambert et al. (2007) and Leuz & Wysocki (2008) for some examples. My assumption is that this holds true on average, but does not always have to be the case.

[25] O'Brien (1988), Abarbanell (1991), Mendenhall (1991), Schipper (1991), Francis & Philbrick (1993), Abarbanell & Lehavy (2003), Easton & Sommers (2007), Larocque (2013), So (2013), Kothari et al. (2016) and Grinblatt et al. (2018) are examples for studies that discuss and analyze analyst bias. Feltham & Ohlson (1995), Zhang (2000) and Skogsvik & Juettner-Nauroth (2013) discuss the effect of conservatism on valuation.

Frequentists have been grappling with these issues, and little has been put on the table. There are studies that concentrate on the ex-ante correction or prediction of analyst forecast errors (see So (2013) for an example), endeavours to model conservatism (see Basu (1997) and Khan & Watts (2009) for examples) and efforts to separate discretionary from non-discretionary accruals. These solutions are, typically, impractical and hardly aid investors in the decision-making process for individual firms.[26] The problem with these sources of biases is that they can only be vaguely quantified ex-ante in a frequentist sense (conservatism and discretionary accruals more so than forecast errors) and leave a lot of room for subjective interpretation in each individual case. Formally, that means that they cannot be objectively included in p(D|H).

Bayesians treat accounting information as a set of signals meant to alter beliefs of investors through the likelihood function; a signal sent by the firm through its financial statements is not more than an information event that is used in the forecasting of payoffs.[27] The important distinction from frequentist thinking is that informative signals under Bayesian rationale do not have to be reflective of true accounting numbers; they can be biased and have higher value in the decision-making process because Bayesians focus on utility:

> *"Even the fact that an estimator is unbiased is not always desirable of itself. Rather, the estimate with highest expected utility, when it is used to revise beliefs and make a decision, can sometimes be a very biased estimator in the frequentist sense."* – Johnstone (2018).

In turn, biases like conservatism can have a positive impact if they lead to increased decision-usefulness of information. Bayesian analysis allows investors to intuitively incorporate vague and hard-to-quantify concepts like conservatism by adjusting the likelihood function p(D|H) and p(H) accordingly. This might be seen as too subjective from a frequentist perspective, but it is ultimately necessary. Especially in the context of these biases however, it is important to avoid overconfidence in the likelihood function or priors that are too informative.

Model Uncertainty

For the estimation of intrinsic value as an unknown parameter, investors cannot know ex-ante which valuation model best measures intrinsic value. Cremers (2002), Avramov & Zhou (2010) and Breuer & Schütt (2021) point out that frequentists tend to ignore model uncertainty because they pick a single model with the highest goodness-of-fit statistics (e.g. adjusted R^2) and act as if the chosen model is the true model. In probabilistic terms, goodness-of-fit statistics determine p(D|M), where M is selected based on the maximum p(D|M) out of all fitted models. As a solution, Bayesians typically suggest the application of model averaging, i.e. including the model as a random variable that is subject to uncertainty as well.[28] This results in the probability p(M|D), allowing a direct comparison of competing models.

But: intrinsic value is not observable and in turn, validating a model for it with certainty is impossible. The validation can only be approximated through stock returns, but it is impossible to rule out that stock returns are driven by omitted variables, noise or intervening news. Because of this issue, it seems reasonable that investors

[26] For example, Khan & Watts (2009) measure conservatism with a metric that requires market-wide panel data, has limited utility as independent variable in other models due to omitted variables-problems and is designed for US-GAAP only.

[27] Signals can be seen as exogenous and/or endogenous; Gao (2013a), Gao (2013b) and Stocken (2013) describe that characterizing signals coming from firms as purely exogenous may not be sufficient if the receiver can influence the signal. Most fundamental investors are rather small scale and have no significant influence on the firm, so I treat signals (and thus also entry prices) as purely exogenous in this paper.

[28] Hinne et al. (2020) provide a detailed guide for practical application of Bayesian model averaging.

may rely on conceptual factors to select a suitable valuation model through the prior p(M) or apply model averaging for practical valuation.

Parameter uncertainty

In a frequentist world, parameters are fixed and unchanging quantities. In turn, there is no uncertainty about the parameters themselves, they are treated as constants that the investor tries to estimate with as much data as possible.[29] Therefore, parameters do not have a probability distribution and one can only determine p(D|P), where P is the parameter of choice. In practice however, capital markets evolve, intrinsic values change constantly; ignoring parameter uncertainty – as the frequentist approach does – implies a level of certainty that is simply not realistic. The Bayesian interpretation of parameters treats them as random variables that may change, just as the world does. This probabilistic understanding of parameters requires a transparent consideration of parameter uncertainty through p(P|D). Finding the exact and true intrinsic value for a firm is impossible because it is "*elusive*" and changes constantly:

> *"Intrinsic value is therefore dynamic in that it is a moving target which can be expected to move forward, but in a much less volatile manner than typical cyclical or other gyrations of the market."* – Graham & Dodd (1934)

The important inference drawn from this subtle quality of intrinsic value is that parameter uncertainty (similar to model uncertainty) cannot be reduced to zero, even if investors had access to all publicly available information and incorporated it rationally and efficiently.[30] Since the future is uncertain and intrinsic values evolve, the only thing investors can do is to make educated guesses on the probability distribution of intrinsic value based on the information they have available, and the Bayesian view does exactly that through the posterior p(H|D). Accounting for parameter uncertainty in valuation, though, is undoubtedly challenging. Valuation models include a large variety of parameters that are subject to uncertainty, i.e. short-term payoffs, the expected date of convergence, long-term growth, the cost of capital and assumptions on payout.[31] In such complex cases, hierarchical Bayesian models can be used to account for parameter uncertainty. These models often rely on machine learning algorithms for random variables (like Markow-Chain-Monte-Carlo; see Ying et al. (2005), Zuo & Kita (2012) or Zhao et al. (2019) for examples).[32]

News uncertainty

Capital markets force investors to constant discovery of information and revisions of previous decisions. Without prior information, there is no posterior of other studies that can be used in subsequent research projects to revise what has been learned before. In practice, learning is not one-off; investors do not discard all they know about a firm after they made the decision to (or not to) invest. Instead, investors monitor the stock price after their decision and upon the arrival of new signals that are perceived as decision-useful, they use the previous posterior to revise the beliefs on the future economic performance:

[29] See Giaquinto et al. (2014) and Midway (2019) for more detailed discussions on the topic.
[30] This is also pointed out by Lewellen & Shanken (2002) in a seminal paper.
[31] Yee (2008b) lists convergence uncertainty as a separate source of uncertainty, but in a Bayesian sense, it can be seen as a type of parameter uncertainty, where the date of convergence is a random parameter.
[32] Unfortunately, hierarchical models – due to the large number of parameters on multiple levels to condition upon – are quite hard to understand and interpret for investors. In practice, investors need to decide between parsimony and accounting for uncertainty.

"The learning process rarely, if ever, discards previous knowledge and replaces it with something completely different. Our information processing is then 'Bayesian'. The Bayesian approach, which is assumed in economics of all rational inference and choice, is consistent with adjusting our statistical analysis to better align with the way our natural mental processing tends to update and incorporate new information." – Dyckman (2016).

Such revisions of previous positions require the possibility to update the "old" posterior probability p(H|D) using new data. To do that, it is necessary to treat the "old" posterior as the "new" prior via p(H') = p(H|D), because only then both H and D can be updated simultaneously. NHST for example only tests the statistical significance of the "new" D, but suggests no alternative for the old H.

3.4. Quo Vadis Equity Valuation?

Graham & Dodd (1934) advise investors to apply an additional discount to the estimate of intrinsic value to account for the uncertainty of estimation: the Margin of Safety. While they do not explicitly mention or apply Bayesian inference, their book suggests many principles and tenets that align well with it.[33] An example for a well-fitting element of their investment philosophy is Cromwell's rule, which states that there can (almost) never be a prior probability of 0 or 1 for an event in Bayesian terms (so that $0 < p(H) < 1$ must hold for all H), because that would imply ultimate certainty and force posterior distributions to be equal to the prior.[34] In consequence, finding the exact intrinsic value of a firm (or a long-run payoff used for finding it) is impossible; the prior probability of the intrinsic value of Amazon being exactly 50 $ can never be 100%. Point estimates for expected payoffs or intrinsic value – as often used in equity valuation – imply that the probability for said estimate is 1, neglecting the high inherent uncertainty.[35] Bayesian thinking fits perfectly to the Margin of Safety, since it is a (subjective) threshold of the probability of intrinsic value exceeding (or falling short of) the market price in a posterior distribution.
Overall, the Bayesian approach has several arguments on its side because it allows investors to make subjective probability assessments under uncertainty and thus suits decision-making on capital markets. As a summary of the discussion, Table 1 shows a comparison of the "Bayesian" investor demands and the frequentist approach for the respective characteristic of the inference problem.

Table 1:

Comparison of "Bayesian" Investor Demands with Frequentist Approach

Characteristic of the Inference Problem	Bayesian Investor Demands	Frequentist Approach
Understanding of Probability	Estimation of conditional probability of hypotheses via p(H|D)	Estimation of conditional probability of the data via p(D|H)

[33] It is worth mentioning that back then, it was basically impossible to conduct computationally demanding Bayesian analyses due to technological constraints.

[34] Cromwell's rule was defined by Lindley (1985), who stated that one should "*leave a little probability for the moon being made of green cheese; it can be as small as 1 in a million, but have it there since otherwise an army of astronauts returning with samples of the said cheese will leave you unmoved.*" Exceptions of the rule apply to expressions that are logically false or to continuous distributions, where the probability for a particular value is always 0.

[35] Often, sensitivity analyses are used to alleviate the issue, but only to some degree. In practice, these analyses require the investor to assign certain probabilities to the respective scenario, so one might as well use a Bayesian approach right away. Another way to account for uncertainty would be to increase the discount rate, but commonly used methodologies from finance (e.g. CAPM) do not allow for such flexible adjustment of the discount rate.

Inclusion of Subjective Information	Accounting for information outside of D and exclusion of „economic nonsense” via P(H)	Accounting for information outside of D indirectly through H, no exclusion of “economic nonsense”
Handling of Information Uncertainty	Inclusion of biases via p(H), which also affects p(H|D)	Exclusion of hard-to-quantify biases, since p(H) does not exist
Handling of Model Uncertainty	Treatment of the model as an additional random parameter that is subject to uncertainty through p(M|D)	Choice of a „true“ model through p(D|M)
Handling of Parameter Uncertainty	Treatment of parameters as random variables with probability distributions via p(P) and p(P|D)	Parameters are fixed unknowns without a probability distribution that can be found via p(D|P)
Handling of News Uncertainty	Revision of previous decisions via new prior p(H‘) and old posterior p(H|D), so that both H and D are updated	Revision only possible through changes in statistical significance via p(D|H), so that only D is updated

4. *Derivation of a Bayesian Accounting-Based Equity Valuation Model*

4.1. Linking Expected Utility to Stock Returns

I strive to provide investors with a valuation model that…

a) supports the decision-making process under uncertainty by allowing assessments of expected utility and

b) links intrinsic value to future expected payoffs through quality drivers and is supported both conceptually and empirically.

Before I start with the model derivation, I want to note that I do not consider Bayesian model averaging in this derivation. While it is certainly useful for a representation of model uncertainty, it drastically reduces the parsimony of the application and theoretically, all discounted payoff valuation models are mathematically equivalent. For the same reason, I do not explicitly model information uncertainty and news uncertainty here. Modelling information uncertainty for example requires a concrete understanding of what kind of information is decision-useful and how it affects both prior and likelihood in the model.[36] Overall, the model I provide can be seen as the architecture for Bayesian inference in equity valuation.

The starting point of the derivation anchors on $\widetilde{U}$, the investors utility relative to the investment – which is assumed to be equal to the expected financial gain or loss from the investment over a particular period in time. This can be expressed through the total stock return (TSR) as follows:

$$\widetilde{U} = \widetilde{TSR}_1^{I,0} = \frac{\widetilde{D}_1}{P_0} + \frac{\widetilde{P}_1^{I,0} - P_0}{P_0}, \qquad (3)$$

where $\widetilde{TSR}_1^{i,x}$, $\widetilde{D}_1^{i,x}$ and $\widetilde{P}_1^{i,x}$ reflect the investor’s $(i = I)$ current $(x = 0)$ estimates of the total stock return and dividend for the convergence window and the current estimate of the future market price after the convergence window, all assumed to be random variables, as indicated by the tilde. A key (and often hidden) assumption of

[36] Johnstone (2015) provides more insights on the matter. Similar arguments can be made for news uncertainty, to an even stronger degree, since then, the set of all available information is also unknown.

fundamental analysis (especially in textbooks on the matter) is that the market price converges to intrinsic value, so that $\widetilde{\mathrm{P}}_1^{M,0} = \widetilde{\mathrm{V}}^{\mathrm{I},0}$, with $\widetilde{\mathrm{V}}^{\mathrm{I},0}$ being the investor's current estimate of intrinsic firm value. In essence, this assumption implies that intrinsic value leads price, which results in the following equation.

$$\widetilde{\mathrm{U}} = \widetilde{\mathrm{ISR}}_1^{\mathrm{I},0} = \frac{\widetilde{\mathrm{D}}_1^{\mathrm{I},0}}{\mathrm{P}_0} + \frac{\widetilde{\mathrm{V}}^{\mathrm{I},0} - \mathrm{P}_0}{\mathrm{P}_0},$$

This equation alters the understanding of the variables of interest so that the intrinsic stock return $\widetilde{\mathrm{ISR}}_1^{\mathrm{I},0}$ becomes a random variable that depends on the distributions of both forward dividends and intrinsic value, instead of being a fixed unknown parameter. This does not mean that intrinsic value is unpredictable, but rather that it cannot be predicted with ultimate certainty. It also implies that the utility from the investment is random and cannot be assessed with certainty, since it is determined by multiple random variables. The convenient quality of treating the essential valuation inputs as random variables (opposed to fixed unknowns) is that investors can quantify the uncertainty of estimation through statements on the actual probability distributions of the inputs.[37]

4.2. Closing in on Quality-Related Information

So far, the model is intuitive, but there is no link of $\widetilde{\mathrm{V}}^{\mathrm{I},0}$ to a particular set of payoffs that can then be filled with life in form of probability distributions. As Lee (2015) & Huefner & Rueenaufer (2021) show, the residual income model (RIM) provides elegant links to payoffs that connect to a stock's inherent degree of quality. Also, there is empirical evidence of the model's usefulness in explaining stock return patterns, e.g. Frankel & Lee (1998), Asness et al. (2019), Li & Mohanram (2019) or Huefner & Rueenaufer (2021). The basic model based on Ohlson (1995) and Feltham & Ohlson (1999) looks as follows:

$$\mathrm{P}_0 = \mathrm{B}_0 + \sum_{\mathrm{t}=1}^{\infty} \frac{(\mathrm{ROE}_\mathrm{t} - \mathrm{r}_\mathrm{t})\mathrm{B}_{\mathrm{t}-1}}{(1+\mathrm{r}_\mathrm{t})}, \tag{4}$$

where P_0 is the equity market price, B_t is the book value of equity, E_t are earnings for the period and r_t is the discount rate (or cost of equity capital) for the respective period. This modal requires the assumption of clean-surplus-accounting (CSA) to hold: $\mathrm{B}_\mathrm{t} = \mathrm{B}_{\mathrm{t}-1} * \left(1 + \mathrm{ROE}_\mathrm{t}^{\mathrm{I},0}(1 - \mathrm{k}_\mathrm{t}^{\mathrm{I},0})\right)$. I further assume that the equity value $\widetilde{\mathrm{V}}^{\mathrm{I},0}$ and the prospective parameters $\tilde{\mathrm{r}}_\mathrm{t}^{\mathrm{I},0}$, $\tilde{\mathrm{k}}_\mathrm{t}^{\mathrm{I},0}$ and $\widetilde{\mathrm{ROE}}_\mathrm{t}^{\mathrm{I},0}$ are random variables with unknown probability distributions:

$$\widetilde{\mathrm{V}}^{\mathrm{I},0} = \mathrm{B}_0 + \sum_{t=1}^{\infty} \left(\frac{(\widetilde{\mathrm{ROE}}_\mathrm{t}^{\mathrm{I},0} - \tilde{\mathrm{r}}_\mathrm{t}^{\mathrm{I},0})\mathrm{B}_{\mathrm{t}-1}}{(1+\tilde{\mathrm{r}}_\mathrm{t}^{\mathrm{I},0})^t} \right). \tag{5}$$

Then, CSA can be expressed through $\mathrm{B}_\mathrm{t} = \mathrm{B}_{\mathrm{t}-1} * \left(1 + \widetilde{\mathrm{ROE}}_\mathrm{t}^{\mathrm{I},0}(1 - \tilde{\mathrm{k}}_\mathrm{t}^{\mathrm{I},0})\right)$, where $\widetilde{\mathrm{ROE}}_\mathrm{t}^{\mathrm{I},0}(1 - \tilde{\mathrm{k}}_\mathrm{t}^{\mathrm{I},0})$ represents the growth rate of book value over the respective period t. Since infinite detailed forecasting is not practical, I truncate

[37] Winkler (1973) and Winkler & Barry (1975) perform a similar derivation, but with a focus on cross-sectional portfolio selection instead of intrinsic value.

the model focus on the expectational values of the three prospective parameters, so that the model can be expressed as a simple perpetuity. With the assumption of growth in residual income equalling growth in book value in the future, the final model equation looks as follows:

$$\tilde{V}^{I,0} = B_0 + B_0 \left(\frac{\left(E[\widetilde{ROE}_t^{I,0}] - E[\tilde{r}_t^{I,0}]\right)}{E[\tilde{r}_t^{I,0}] - E[\widetilde{ROE}_t^{I,0}]\left(1 - E[\tilde{k}_t^{I,0}]\right)} \right). \quad (6)$$

Here, $E[\tilde{r}_t^{I,0}]$ is expected long-run cost of equity capital, $E[\widetilde{ROE}_t^{I,0}]$ is the expected long-term ROE and $E[\tilde{k}_t^{I,0}]$ is the long-term payout ratio, as currently expected by the investor. This model requires going-concern, transversality and clean-surplus accounting (CSA) and expresses the current equity value as the sum of terms that differ in the uncertainty of estimation. It is important to mention that this equation does not allow investors to make inferences about the expectational values of intrinsic value, future book values, dividends and earnings due to Jensen's inequality.[38] Since my endeavour strives to account for uncertainty through the posterior probability distributions of $E[\tilde{r}_t^{I,0}]$, $E[\tilde{k}_t^{I,0}]$ and $E[\widetilde{ROE}_t^{I,0}]$, expectational values for these parameters are not the main concern anyway. For simplicity of notation, $E[\tilde{r}_t^{I,0}]$, $E[\tilde{k}_t^{I,0}]$ and $E[\widetilde{ROE}_t^{I,0}]$ are labelled as $\tilde{r}_L^{I,0}$, $\tilde{k}_L^{I,0}$ and $\widetilde{ROE}_L^{I,0}$ from here onwards.

5. Guidance on the Prior, Likelihood and Posterior Distributions of Long-Term Parameters

For illustration, I apply the considerations and calculations shown in this section to a practical example along the way. The data required for the simulation is obtained from public sources (Yahoo Finance) as of 22nd January 2022. The example used is the German automobile manufacturer Volkswagen, chosen randomly out of all companies listed on the German stock market. For the likelihood functions in the example, I use forecasts for earnings and dividends over the next three years in the form of mean consensus analyst forecasts and estimate the likelihood function of beta using daily returns from both Volkswagen and the S&P 500 Index for the last 10 trading days prior to the estimation. The data and code for the replication of the calculations and figures included in this section can be found on the author's GitHub-page. Following the central limit theorem, all averages of random variables are asymptotically normally distributed with an increasing number of observations, regardless of the distribution of the random variable. Using this, I employ a normal-normal model for the long-term parameters because these can be seen as long-term averages that represent the infinite series of future ROEs, discount rates and payout ratios.[39] Given that the expected number of observations for these variables is infinite (if only in expectation), the normality assumption seems reasonable. Formally, let $\tilde{r}_L^{I,0}$, $\tilde{k}_L^{I,0}$ and $\widetilde{ROE}_L^{I,0}$ be random variables $\tilde{\alpha}_j$ that adhere to the following equation:

$$\tilde{\alpha}_j = \mu_{\tilde{\alpha}_j} + \sigma_{\tilde{\alpha}_j}\tilde{\omega}, \quad (7)$$

here $\mu_{\tilde{\alpha}_j}$ is the mean of the prior distribution, $\sigma_{\tilde{\alpha}_j}$ is the factor scaling the standard deviation and $\tilde{\omega}$ follows a standard normal distribution such that $\tilde{\omega} \sim N(0,1)$. Alternative assumptions on the respective distribution are of

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[38] Precisely, Jensen's inequality states that $E[f(\tilde{X})] \geq f(E[\tilde{X}])$, where $\tilde{X}$ is a random variable. For the example of the random variable $\tilde{B}$ (for book value), it is true that $1/E[\tilde{B}] \neq E[1/\tilde{B}]$, which is why $E[\widetilde{ROE}] = E[\tilde{E}/\tilde{B}] \neq E[\tilde{E}]/E[\tilde{B}]$ must hold. See Kruschwitz & Löffler (2020) for another demonstration in valuation models and Grinblatt & Linnainmaa (2011) for further insights.

[39] Taking the cost of equity as an example, this assumption implies that $\tilde{r}_L^{I,0} = \frac{r_t + r_{t+1} + \cdots + r_n}{n}$, where $r_t, r_{t+1} \ldots, r_n$ are random draws from $\tilde{r}^{I,0}$.

course possible. $\mu_{\tilde{\alpha}_j}$ and $\sigma_{\tilde{\alpha}_j}$ differ across variables and depend on the exact prior and likelihood distributions suggested by economic constraints, valuation theory and the data at hand. The model includes several mathematical operations with normally distributed variables, so the output is not a normally distributed variable, at least not under the conditions in this analysis.[40] Using normal-normal models also entails a set of assumptions that are critical limitations in cases where they are not applicable, namely that observations are uncorrelated and that the population variance is known.[41] Treating the variance as unknown and including potential correlations across observations is more realistic, but also complicates the model application.

The model anchors on three prospective random variables – $\tilde{r}_L^{I,0}$, $\tilde{k}_L^{I,0}$ and $\widetilde{ROE}_L^{I,0}$ – while the anchoring book value will be treated as a constant. Despite its simplicity, the dependence of the three random variables within the model requires a particular stepwise, algorithmic procedure for the estimation of intrinsic value.[42] The result is a requirement of simulation techniques (e.g. Markow-Chain-Monte-Carlo), because there is no closed-form solution to the inference problem. The next section will cover the respective prior, likelihood and posterior of the three long-run parameters as well as a suggestion for a complete simulation algorithm for the probability distribution of $\tilde{V}^{I,0}$.

For the normal-normal model, it is necessary to specify what $\mu_{\tilde{\alpha}_j}$ and $\sigma_{\tilde{\alpha}_j}$ for the prior and likelihood of each parameter could be. For the prior, it is useful to look at the assumptions included in the model and examine the implications of the assumptions on the posterior. The two assumptions that define feasible intervals are *going concern* and *transversality*.

- First, *going concern* requires investors to avoid any assumption that implies firm failure (in form of bankruptcy for example). Paired with clean surplus accounting, *going concern* implies that $\widetilde{ROE}_L^{I,0} > 0$ and $0 < \tilde{k}_L^{I,0} < 1$ must hold (or $p[\widetilde{ROE}_L^{I,0}] > 0 = 1$ and $p[\tilde{k}_L^{I,0} \in (0,1)] = 1$). Otherwise, persistent losses and/or a non-sustainable payout policy perpetually drain the book value of equity, leading to firm failure sooner or later.[43]
- Second, *transversality* demands that the denominator in the terminal value is strictly positive, as the series of discounted payoffs diverges otherwise. To fulfil that requirement, both $\tilde{r}_L^{I,0} > 0$ and $\widetilde{ROE}_L^{I,0}(1 - \tilde{k}_L^{I,0}) < \tilde{r}_L^{I,0}$ must hold (or $p(\tilde{r}_L^{I,0}) > 0 = 1$ and $p[\widetilde{ROE}_L^{I,0}(1 - \tilde{k}_L^{I,0}) < \tilde{r}_L^{I,0}] = 1$, respectively).[44]

If one if these conditions is not satisfied, the valuation can yield negative results for intrinsic value. This is a pervasive problem in existing papers performing valuation on a large scale, like Frankel & Lee (1998) or Francis et al. (2000), since they frequently include ad-hoc assumptions that do not fulfil either going-concern or transversality. Going concern and transversality imply that the interval for $\widetilde{ROE}_L^{I,0}$ depends on both $\tilde{r}_L^{I,0}$ and $\tilde{k}_L^{I,0}$, so I discuss the prior, likelihood and posterior for $\tilde{r}_L^{I,0}$ and $\tilde{k}_L^{I,0}$ first and then estimate the conditional distribution of

[40] Hinkley (1969) shows that the distribution of the ratio of two correlated normally distributed variables is only approximately normal if the distribution of the denominator is far enough away from zero and non-negative. Equity valuation often involves denominators close to zero, putting this assumption into question.
[41] For fundamental data such as ROEs or payout ratios, it is probable that observations show autocorrelation, since they are rather persistent, as Nissim & Penman (2001) or Huefner & Rueenaufer (2021) show.
[42] As Penman & Reggiani (2013), Penman & Reggiani (2018) and Penman et al. (2020) argue, it is possible that a long-run growth rate (or ROE in this case) and the cost of equity capital can be related because expected growth is always at risk.
[43] Note that this just requires positive earnings, not positive residual income.
[44] This further implies that $\tilde{g}_L^{I,0} < \tilde{r}_L^{I,0}$, where $\tilde{g}_L^{I,0}$ is the long-term earnings growth rate equal to $\widetilde{ROE}_L^{I,0}(1 - \tilde{k}_L^{I,0})$.

$\widetilde{\text{ROE}}_L^{I,0}$.[45] Paired with the assumption of a non-truncated normal distribution of the parameters, this puts three overarching requirements on the three prior distributions:

R1) the prior mean $\mu_{\underline{\alpha}_j}$ must be within the feasible interval,

R2) the prior standard deviation $\sigma_{\underline{\alpha}_j}$ must be specified so that the chance for values outside of the interval is effectively zero,

R3) the mean and standard deviation should cover the space of the interval as much as possible.

In turn, R2 and R3 are in a trade-off, since a smaller (larger) $\sigma_{\underline{j}}$ increases (decreases) the chance for feasible output, but shrinks (widens) the range of the feasible interval covered by the distribution. Priors that fulfil all three requirements can be considered as weakly informative in the context of the valuation task.[46] In the following part of this section, I review existing literature on the prior and likelihood distributions of the three variables with a consideration of the abovementioned requirements.

5.1. Prior, Likelihood and Posterior for the Cost of Equity Capital

The cost of equity capital is – according to Kruschwitz & Löffler (2020) – "one of the key concepts in finance". As shown by Pinto et al. (2019), the most prevalent model in professional valuation practice is the Capital Asset pricing Model (CAPM) based on Sharpe (1964). Despite its flaws, one of the major benefits of the model is that it is very straightforward to execute. More importantly though, several contributions to the field – e.g. Vasicek (1973), Shanken (1990), Karolyi (1992) and Cosemans et al. (2016) – suggest that the model's estimates improve noticeably when additional information is incorporated through a prior; the estimates are more reliable and have higher predictive power for returns.[47] Anchoring on Sharpe (1964) and using $\tilde{r}_L^{I,0}$, a Bayesian variant of the CAPM can be expressed as follows:

$$\tilde{r}_L^{I,0} = rf^{I,0} + \tilde{\beta}_0^{I,0} MRP^{I,0} + \tilde{\varepsilon}, \tag{8}$$

Where $rf^{I,0}$ is the risk-free interest rate, $\tilde{\beta}_0^{I,0}$ is the investors estimate of the current firm beta, $MRP^{I,0}$ is the market risk premium over $rf^{I,0}$ and $\tilde{\varepsilon}$ is the residual error term. The simplifying assumption included here is that the cost of equity capital is time-invariant, but randomly drawn from a particular probability distribution. Since both $rf^{I,0}$ and $MRP^{I,0}$ are market wide and assumed to be strictly positive, only the factor loading $\tilde{\beta}_0^{I,0}$ and the error term $\tilde{\varepsilon}$ – as the two random variables on the RHS – are responsible for the cross-sectional variation of $\tilde{r}_L^{I,0}$. Given that the investor's posterior is $\bar{r}_L^{I,0} \propto N\left(\mu_{\bar{r}_L^{I,0}}, \sigma_{\bar{r}_L^{I,0}}\right)$, $\bar{\beta}_0^{I,0}$ must also be normally distributed, so that $\mu_{\bar{r}_L^{I,0}} = rf^{I,0} + \mu_{\bar{\beta}_0^{I,0}} MRP^{I,0}$ and $\sigma_{\bar{r}_L^{I,0}} = \sigma_{\bar{\beta}_0^{I,0}} MRP^{I,0}$. The variables of interest here are $\mu_{\bar{\beta}_0^{I,0}}$ and $\sigma_{\bar{\beta}_0^{I,0}}$, for which prior and likelihood are needed. Since $\bar{r}_L^{I,0}$ is (proportionally) normally distributed, both prior and likelihood are normally distributed as well. Accordingly, I define the investor's prior for $\tilde{r}_L^{I,0}$ as $\underline{r}_L^{I,0} \sim N\left(\mu_{\underline{r}_L^{I,0}}, \sigma_{\underline{r}_L^{I,0}}\right)$ and the likelihood as $\hat{r}_L^{I,0} \sim N\left(\mu_{\hat{r}_L^{I,0}}, \sigma_{\hat{r}_L^{I,0}}\right)$. The likelihood here includes the standard OLS-based beta, which combines with the prior to

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[45] One could, of course, change the direction of this dependence and estimate the distributions in a different order. I explain why I chose this order later on.

[46] See Gabry et al. (2019) for a more detailed outline of weakly informative priors and their characteristics.

[47] Alternative models to the CAPM can be tested in future research, e.g. multi-factor models based on Fama & French (1992), Fama & French (1995) and Carhart (1997) or accounting betas based on Nekrasov & Shroff (2009).

the posterior distribution. Vasicek (1973) describes a shrinkage estimator for the posterior mean of beta that combines the normal prior and likelihood as follows:

$$\mu_{\overline{\beta}_0^{I,0}} = \frac{\mu_{\underline{\beta}_1^{I,0}} / \sigma^2_{\underline{\beta}_0^{I,0}} + \mu_{\hat{\beta}_1^{I,0}} / \sigma^2_{\hat{\beta}_0^{I,0}}}{1/\sigma^2_{\underline{\beta}_0^{I,0}} + 1/\sigma^2_{\hat{\beta}_0^{I,0}}}. \tag{9}$$

Accordingly, the posterior variance is equal to

$$\sigma^2_{\overline{\beta}_0^{I,0}} = \frac{1}{1/\sigma^2_{\underline{\beta}_0^{I,0}} + 1/\sigma^2_{\hat{\beta}_0^{I,0}}}. \tag{10}$$ [48]

These values can then be used to obtain the posterior mean and variance of $\tilde{r}_L^{I,0}$:

$$\mu_{\overline{r}_L^{I,0}} = rf^{I,0} + \overline{\beta}_0^{I,0} MRP^{I,0} \; \& \; \sigma^2_{\overline{r}_L^{I,0}} = \sigma^2_{\overline{\beta}_0^{I,0}} (MRP^{I,0})^2. \tag{11}$$

For the determination of $\mu_{\underline{\beta}_0^{I,0}}$, the prior mean, several strategies exists and most of them use OLS-based methods. In Bayesian applications, a shrinkage-based approach is the most common. Vasicek (1973) suggests that if one knows nothing about a stock, it is reasonable to assume that the stock moves exactly with the market, so that $\mu_{\underline{r}_L^{I,0}} = rf^{I,0} + MRP^{I,0}$ because of $\mu_{\underline{\beta}_0^{I,0}} = 1$. Karolyi (1992) argues that the market-based prior is a good starting point, but often ignores available information, since information on e.g. firm industry is often known prior to the estimation. Cosemans et al. (2016) use a model based on Shanken (1990) that anchors on fundamental firm data to derive more sophisticated priors for the beta. Out of these strategies, only the market-wide prior effectively ensures that $p[\underline{r}_L^{I,0}] > 0 \approx 1$ because both industry betas and betas based on fundamentals can lead to $\underline{\beta}_0^{I,0} < 0$, so R1 might be violated.

For the prior standard deviation of beta, Vasicek (1973) employs the cross-sectional unconditional standard deviation. In turn, the prior distribution treats each firm as a random draw from the cross-section. This approach is consistent with the market wide prior mean, but does not guarantee that R2 is fulfilled. As a remedy, I suggest the following procedure for the determination of $\sigma_{\underline{r}_L^{I,0}}$ and $\sigma_{\underline{\beta}_0^{I,0}}$:

1. calculate the difference between prior mean and the risk-free rate,
2. set the probability threshold z for prior with $p\left(\underline{r}_L^{I,0} > rf^{I,0}\right) \approx (1 - z)$,
3. reverse engineer the prior standard deviation $\sigma_{\underline{r}_L^{I,0}}$ through $\Phi\left[\left(\mu_{\underline{r}_L^{I,0}} - rf^{I,0}\right)/\sigma_{\underline{r}_L^{I,0}}\right] \approx z$ using the cumulative density function for the standard normal distribution[49] and
4. obtain $\sigma_{\underline{\beta}_0^{I,0}}$ through $\sigma_{\underline{\beta}_0^{I,0}} = \sigma_{\underline{r}_L^{I,0}} / MRP^{I,0}$.

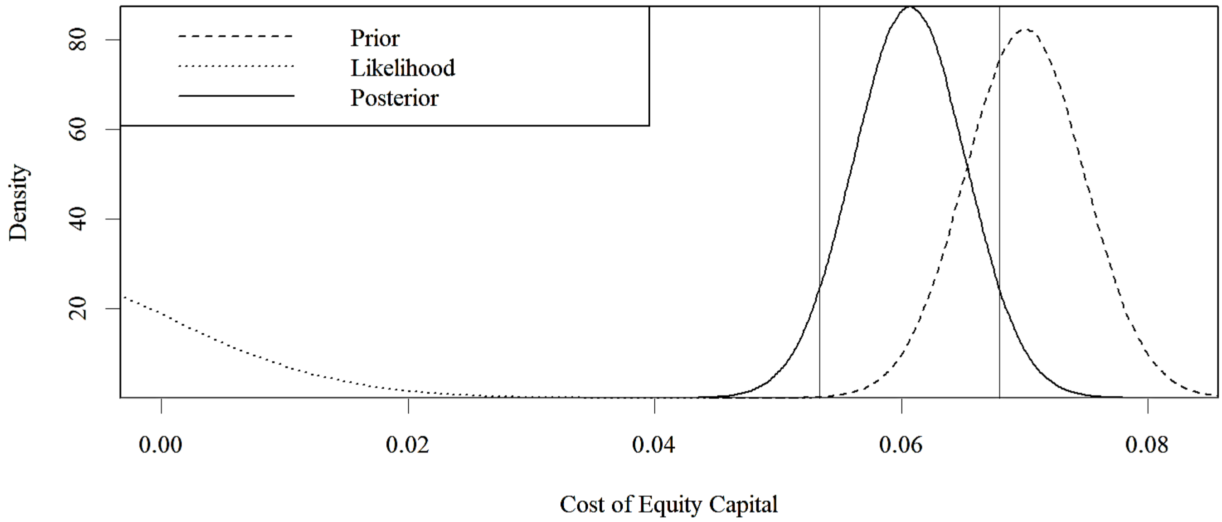
[48] This estimator simplifies the weighting across prior and likelihood, because it does not include the sample size (i.e. the amount of evidence) used in the estimation on the likelihood.

[49] The exact notion of the cumulative density function for the normally distributed parameter is $F\left(\underline{r}_L^{I,0}\right) = \Phi\left[\left(\mu_{\underline{r}_L^{I,0}} - rf^{I,0}\right)/\sigma_{\underline{r}_L^{I,0}}\right] = 1/2\left[1 + \mathrm{erf}\left(\left(\mu_{\underline{r}_L^{I,0}} - rf^{I,0}\right)/\sigma_{\underline{r}_L^{I,0}}\sqrt{2}\right)\right]$, with $\mathrm{erf}\left(\underline{r}_L^{I,0}\right) = 2/\sqrt{\pi}\int_{-\infty}^{\pi} e^{-t^2}$. This function applies analogously to priors, likelihoods and posteriors of other parameters.

For the weakly informative prior, z must be set close enough to one to fulfil R2, but also not eliminate too much of parameter uncertainty so that R3 is fulfilled as well. This way, the prior standard deviation always scales with the feasible interval on a firm-specific level and with a small enough z, rules out that random draws from the posterior land outside of the feasible interval.

Figure 2:

Prior, Likelihood and Posterior Distribution for the Cost of Equity Capital ($\tilde{r}_L^{I,0}$)



For Volkswagen, I set z to 0.1[35] to ensure that $\left(\tilde{r}_L^{I,0}\right) > rf^{I,0}\ \approx 1$ holds. Using the market prior of 1 for $\mu_{\underline{\beta}_0^{I,0}}$ and applying the four steps of the prior variance calculation leads to the prior distribution $\underline{r}_L^{I,0} \sim N(0.07, 0.0048)$. The likelihood values stem from an application of the OLS-CAPM and are equal to $\hat{r}_L^{I,0} \sim N(-0.1293, 0.0136)$. Using Equations (9), (10) and (11) then results in a posterior distribution of $\bar{r}_L^{I,0} \propto N(0.0607, 0.0046)$. Figure 2 shows a plot of the three density functions. It is visible that the likelihood is frequently negative; the probability distribution of OLS-based beta included in the likelihood thus largely falls outside the feasible interval. Since the standard deviation of the prior is smaller than the standard deviation of the likelihood, the posterior anchors on the prior and consistently falls into the feasible interval for $\tilde{r}_L^{I,0}$. It is also noteworthy that the posterior still covers a fairly wide range, with an 89% equal-tailed credible interval (CI) of [0.0534, 0.0679]. So based on the data at hand and the weakly informative prior, the investor is only 89% certain that the cost of equity capital for Volkswagen is between 5.34% and 6.79%. Figure 2 shows a plot of the three distributions, with the 89% CI for the posterior included as vertical lines.

5.2. Prior, Likelihood and Posterior for the Long-Term Payout Ratio

Analogous to the cost of capital, I define the prior distribution of the long-term payout ratio as $\underline{k}_L^{I,0} \sim N\left(\mu_{\underline{k}_L^{I,0}}, \sigma_{\underline{k}_L^{I,0}}\right)$. Studies on the payout behaviour of firms are scarce and to the author's knowledge, there is no explicit research on the prior distribution of long-term payout ratios. Since dividends are assumed to be value-irrelevant in most applications of valuation models – as suggested by Modigliani & Miller (1961) – that is not surprising. Among others, Huefner & Rueenaufer (2020) provide arguments for why dividends should not be seen as irrelevant to value under uncertainty, heterogenous expectations and information asymmetry, so I consider them here.

The pioneering empirical study on the long-run payout behaviour of firms is Lintner (1956), who proposed a partial-adjustment model for dividends and showed that dividends are rather persistent and closely tied to earnings and earnings volatility over time. More recent studies – e.g. Fama & French (2001), Grullon & Michaely (2002) and Brav et al. (2005) – demonstrate that share repurchases substitute for dividends more and more, while total payout (including both dividends and share repurchases as a fraction of earnings) increased. This effect should be considered to not understate total payout, even though it is methodologically challenging, as Ohlson (2005) shows. Moreover, Huefner & Rueenaufer (2021) demonstrate that total payout ratios converge in the long run; low quantiles converge upwards, high quantiles downwards.

These studies concentrate solely on the United States; international evidence is provided by Glen et al. (1995), La Porta et al. (2000), Desai & Jin (2011), Alzahrani & Lasfer (2012) and Akhtar (2018). They show that many different factors other than earnings influence payout policy (e.g. industry, country of origin, domestic/multinational orientation, shareholder structure or tax incentives). It is also visible that total payout ratios averaged between ~20% and ~70% in the last two decades. Seeing the empirical evidence, it seems that 50% – the middle of the interval – could be a reasonable starting point for a general prior mean that satisfies R1, and that the abovementioned insights from research can be used for further refinement.

Several studies do not report the standard deviations or variances of the averages they present (e.g. Grullon & Michaely (2002) and Akhtar (2018)), so the reliability as prior information for the standard deviation might be limited. In consequence, I suggest using an approach that optimises the trade-off between R2 and R3, similar to the approach for the cost of equity capital. The convenience of choosing 50% as the prior mean is that it maximises the range of the feasible interval compared to alternative prior means due to the symmetric normality assumption, so it is as weakly informative as possible.[50] More generally for all feasible $\mu_{\underline{k}_L^{I,o}}$, I suggest the following procedure for the determination of $\sigma_{\underline{k}_L^{I,o}}$:

1. calculate the difference between prior mean and the closest distance to either interval border $\tilde{\mu}_{\underline{k}_L^{I,o}} = \min\begin{pmatrix}\mu_{\underline{k}_L^{I,o}} - 0\\ 1 - \mu_{\underline{k}_L^{I,o}}\end{pmatrix}$,
2. set the probability threshold z for prior with $p\left(\underline{k}_L^{I} > \mu_{\underline{k}_L^{I,o}} - \tilde{\mu}_{\underline{k}_L^{I,o}}\right) \approx (1 - z)$ and
3. reverse engineer the prior standard deviation $\sigma_{\underline{k}_L^{I,o}}$ through $\Phi\left[\left(\tilde{\mu}_{\underline{k}_L^{I,o}} - \left(\mu_{\underline{k}_L^{I,o}} - \tilde{\mu}_{\underline{k}_L^{I,o}}\right)\right) / \sigma_{\underline{k}_L^{I,o}}\right] \approx z$ using the cumulative density function for the standard normal distribution.

Given that long-run payout ratios are empirically elusive, estimating the likelihood function $\hat{k}_L^{I,0} \sim N\left(\mu_{\hat{k}_L^{I,o}}, \sigma_{\hat{k}_L^{I,o}}\right)$ is challenging. Fundamental investors place more weight on short-term estimates and historical fundamentals compared to long-term speculation. Textbooks and papers on fundamental analysis include a variety of investment tenets that anchor on this principle, e.g.:

- "***Value gravitates towards fundamentals***, *but it may take some time*" – Penman (2011),
- *"Stocks selling well below the levels apparently justified by* ***a careful analysis of the relevant facts***" – Graham & Dodd (1934) and

[50] This approach is not suitable for the cost of capital because the feasible interval does not have a mean value.

- "***Assign greater weight to the more reliable estimates** when some estimates are more reliable than are others*" – Yee (2008a).

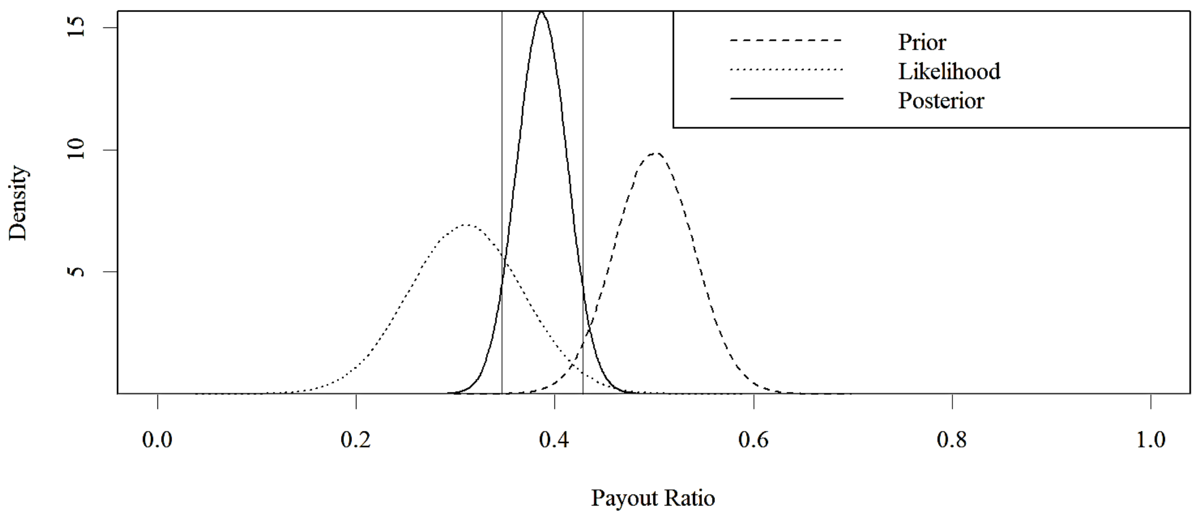
On top of that, there is an large amount of research that links measures of fundamental strength to (abnormally) positive security returns, such as Ou & Penman (1989), Piotroski & So (2012), Asness et al. (2019), Li & Mohanram (2019) or Huefner & Rueenaufer (2021). Another factor of importance demonstrated in these studies is that measures of fundamental quality are typically persistent. The main takeaway for the likelihood of payout ratios is that historical fundamentals and/or short-term estimates can be the foundation for long-term speculation. This way, the likelihood anchors on the actual evidence, so that the posterior mean and variance of long-term payout $\bar{k}_L^{I,0} \propto N\left(\mu_{\bar{k}_L^{I,0}}, \sigma_{\bar{k}_L^{I,0}}\right)$ are results of a weighted combination of prior information and fundamental data. Following Murphy (2007) or Kaplan (2014), the weights for the posterior mean and posterior variance are determined by the number of observations (n) and the variances of prior and likelihood:

$$\mu_{\bar{k}_L^{I,0}} = \sigma^2_{\bar{k}_L^{I,0}} \left(\frac{\mu_{\underline{k}_L^{I,0}}}{\sigma^2_{\underline{k}_L^{I,0}}} + \frac{n\mu_{\hat{k}_L^{I,0}}}{\sigma^2_{\hat{k}_L^{I,0}}} \right) \ \& \ \sigma^2_{\bar{k}_L^{I,0}} = \frac{1}{\frac{1}{\sigma^2_{\underline{k}_L^{I,0}}} + \frac{n}{\sigma^2_{\hat{k}_L^{I,0}}}}. \quad (12)$$

The example prior distribution uses the middle of the feasible interval and the standard deviation based on reverse engineering (again with $z = 0.1^{35}$), resulting in $\underline{k}_L^{I,0} \sim N(0.5, 0.0403)$. For Volkswagen, the expected mean consensus analyst forecasts of payout ratios for the next three fiscal years are 27.05%, 28.43% and 37.63%. Using the mean and standard deviation of these forecasts, the likelihood function for $\tilde{k}_L^{I,0}$ becomes $\hat{k}_L^{I,0} \sim N(0.3104, 0.0575)$. Applying Bayes' theorem to combine them through both parts of Equation (12) yields a posterior distribution of $\bar{k}_L^{I,0} \propto N(0.3872, 0.0256)$. In this case, the posterior shifts more strongly towards the likelihood because the standard deviations of the prior and likelihood are quite similar, and the weight of the likelihood increases with each observation (n = 3). Even though the standard deviation of the posterior seems rather small (with an 89% CI of [0.346, 0.428]), the uncertainty of estimation remains large, as the simulation of

Figure 3

Prior, Likelihood and Posterior for the Long-Term Payout Ratio ($\tilde{k}_L^{I,0}$)



intrinsic value will demonstrate later. Figure 3 shows a plot of the three distributions, again with the 89% CI for the posterior included as vertical lines.

5.3. Prior, Likelihood and Posterior for the Long-Term Return on Equity

Transversality requires that the feasible interval for $\widehat{\mathrm{ROE}}_{\mathrm{L}}^{\mathrm{I},0}$ is set consistently with the posterior distributions of $\tilde{\mathrm{r}}_{\mathrm{L}}^{\mathrm{I},0}$ and $\tilde{\mathrm{k}}_{\mathrm{L}}^{\mathrm{I},0}$ due to the interaction of terminal value components. The solution I propose is a per-draw scenario analysis, so that for each combination of random draws from $\overline{\mathrm{r}}_{\mathrm{L}}^{\mathrm{I},0}$ and $\overline{\mathrm{k}}_{\mathrm{L}}^{\mathrm{I},0}$, a different $\underline{\mathrm{ROE}}_{\mathrm{L}}^{\mathrm{I},0}$ is determined. The main advantage of this approach over choosing e.g. a mean of the variances is that it leads to a posterior that a) is consistently within the feasible interval required by transversality (i.e. the valuation has a smaller probability for "nonsense") and b) accounts for the dependence of ROE on the other two long-run variables.

In order to ensure that R2 and R3 are satisfied, the variance has to be adjusted so that effectively, $\mathrm{p}\left[0 < \widehat{\mathrm{ROE}}_{\mathrm{L}}^{\mathrm{I},0} < \tilde{\mathrm{r}}_{\mathrm{L}}^{\mathrm{I},0}/\left(1-\tilde{\mathrm{k}}_{\mathrm{L}}^{\mathrm{I},0}\right)\right] \approx 1$ for all combinations of draws from $\overline{\mathrm{r}}_{\mathrm{L}}^{\mathrm{I},0}$ and $\overline{\mathrm{k}}_{\mathrm{L}}^{\mathrm{I},0}$. For this reason, generating the (conditional) weakly informative prior for $\widehat{\mathrm{ROE}}_{\mathrm{L}}^{\mathrm{I},0}$ once again anchors on reverse engineering:

1. calculate the difference between prior mean and the closest distance to either interval border $\check{\mu}_{\underline{\mathrm{ROE}}_{\mathrm{L}}^{\mathrm{I},0}} = \min\begin{pmatrix} \mu_{\underline{\mathrm{ROE}}_{\mathrm{L}}^{\mathrm{I},0}} - 0 \\ \overline{\mathrm{r}}_{\mathrm{L}}^{\mathrm{I},0}/\left(1-\overline{\mathrm{k}}_{\mathrm{L}}^{\mathrm{I},0}\right) - \mu_{\underline{\mathrm{ROE}}_{\mathrm{L}}^{\mathrm{I},0}} \end{pmatrix}$, where $\overline{\mathrm{r}}_{\mathrm{L}}^{\mathrm{I},0}$ and $\overline{\mathrm{k}}_{\mathrm{L}}^{\mathrm{I},0}$ represent random draws from the respective posterior distribution,
2. set the probability threshold z for prior with $\mathrm{p}\left(\underline{\mathrm{ROE}}_{\mathrm{L}}^{\mathrm{I}} > \mu_{\underline{\mathrm{ROE}}_{\mathrm{L}}^{\mathrm{I},0}} - \check{\mu}_{\underline{\mathrm{ROE}}_{\mathrm{L}}^{\mathrm{I},0}}\right) \approx (1-\mathrm{z})$ and
3. reverse engineer the prior standard deviation $\sigma_{\underline{\mathrm{ROE}}_{\mathrm{L}}^{\mathrm{I},0}}$ through $\Phi\left[\left(\check{\mu}_{\underline{\mathrm{ROE}}_{\mathrm{L}}^{\mathrm{I},0}} - \left(\mu_{\underline{\mathrm{ROE}}_{\mathrm{L}}^{\mathrm{I},0}} - \check{\mu}_{\underline{\mathrm{ROE}}_{\mathrm{L}}^{\mathrm{I},0}}\right)\right)/\ \sigma_{\underline{\mathrm{ROE}}_{\mathrm{L}}^{\mathrm{I},0}}\right] \approx \mathrm{z}$ using the cumulative density function for the standard normal distribution and
4. repeat steps 1, 2 and 3 process δ times, where δ is the chosen number of randomly generated draws. The larger δ, the more exact the simulation of the distribution will be.

In turn, each δ has a different optimal value for the prior variance, so that simulating the curve for the distribution of intrinsic value requires a consideration of that. For the normal likelihood $\widehat{\mathrm{ROE}}_{\mathrm{L}}^{\mathrm{I},0} \sim \mathrm{N}\left(\mu_{\widehat{\mathrm{ROE}}_{\mathrm{L}}^{\mathrm{I},0}}, \sigma_{\widehat{\mathrm{ROE}}_{L}^{\mathrm{I},0}}\right)$, I apply the same approach as for the payout ratios, so that short-term expected ROEs function as the observations that generate the mean and standard deviation. In contrast to the prior, the likelihood is the same for all δ. This likelihood can then be used together with the respective prior to obtain draws from the different posteriors as follows:

$$\mu_{\delta,\overline{\mathrm{ROE}}_{\mathrm{L}}^{\mathrm{I},0}} = \sigma^2_{\delta,\overline{\mathrm{ROE}}_{\mathrm{L}}^{\mathrm{I},0}} \left(\frac{\mu_{\delta,\underline{\mathrm{ROE}}_{\mathrm{L}}^{\mathrm{I},0}}}{\sigma^2_{\delta,\underline{\mathrm{ROE}}_{\mathrm{L}}^{\mathrm{I},0}}} + \frac{\mathrm{n}\mu_{\widehat{\mathrm{ROE}}_{\mathrm{L}}^{\mathrm{I},0}}}{\sigma^2_{\widehat{\mathrm{ROE}}_{\mathrm{L}}^{\mathrm{I},0}}} \right) \ \&\ \sigma^2_{\delta,\overline{\mathrm{ROE}}_{\mathrm{L}}^{\mathrm{I},0}} = \frac{1}{\frac{1}{\sigma^2_{\delta,\underline{\mathrm{ROE}}_{\mathrm{L}}^{\mathrm{I},0}}} + \frac{\mathrm{n}}{\sigma^2_{\widehat{\mathrm{ROE}}_{\mathrm{L}}^{\mathrm{I},0}}}}. \tag{13}$$

Here, $\mu_{\delta,\underline{\mathrm{ROE}}_{\mathrm{L}}^{\mathrm{I},0}}$ and $\sigma^2_{\delta,\underline{\mathrm{ROE}}_{\mathrm{L}}^{\mathrm{I},0}}$ and $\mu_{\delta,\overline{\mathrm{ROE}}_{\mathrm{L}}^{\mathrm{I},0}}$ and $\sigma^2_{\delta,\overline{\mathrm{ROE}}_{\mathrm{L}}^{\mathrm{I},0}}$ are the respective per-draw mean and variance for the prior and posterior, conditional on $\overline{\mathrm{r}}_{\mathrm{L}}^{\mathrm{I},0}$ and $\overline{\mathrm{k}}_{\mathrm{L}}^{\mathrm{I},0}$.

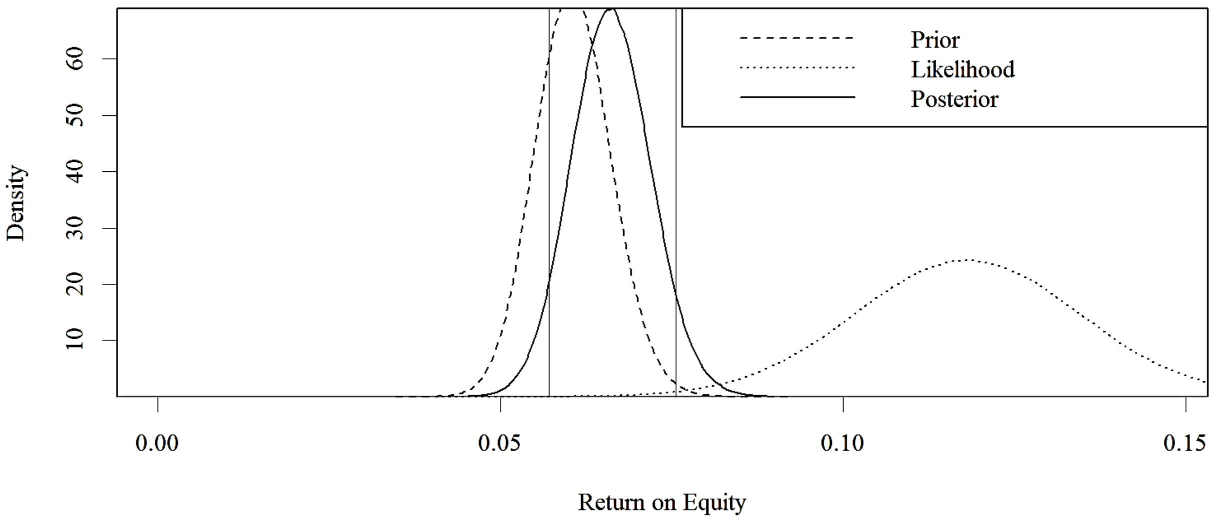
For the determination of the priors $\underline{ROE}_L^{I,0} \sim N\left(\mu_{\delta,\underline{ROE}_L^{I,0}}, \sigma^2_{\delta,\underline{ROE}_L^{I,0}}\right)$, I follow economics of competition. Several papers and textbooks – such as Nissim & Penman (2001), Penman (2013), Palepu et al. (2019) – show that ROEs tend to converge towards industry medians over longer horizons. Investors can use these effects for forecasting, but this approach does not guarantee that R1 is satisfied (since an industry mean or median might be outside the feasible interval). Dechow et al. (1999) demonstrate that residual income converges to zero over time, because competitive advantages are not sustained infinitely. Beaver & Ryan (2000) examine the predictability of ROEs over time and decompose it into a persistent portion and transitory "noise". Similar to Huefner & Rueenaufer (2021), they find that ROEs are somewhat persistent (and thus predictable) over time.

The results of Dechow et al. (1999) are consistent with the impression that – conservatism aside – $\widetilde{ROE}_L^{I,0}$ tends to converge towards $\tilde{r}_L^{I,0}$ in the long run. In turn, $\mu_{\delta,\underline{ROE}_L^{I,0}}$ can be anchored on random draws from $\bar{r}_L^{I,0}$, the posterior of the cost of equity. This is useful because it ensures that R1 is fulfilled in every individual valuation due to $\tilde{r}_L^{I,0} < \tilde{r}_L^{I,0}/(1 - \tilde{k}_L^{I,0})$ for all feasible values of $\tilde{k}_L^{I,0}$. But: accounting conservatism affects future ROEs positively due to understated book values – Zhang (2000) and Skogsvik & Juettner-Nauroth (2013) demonstrate that. Using the cost of capital for $\mu_{\delta,\underline{ROE}_L^{I,0}}$ might therefore be a quite conservative forecast.

For the example calculation, the first step is to draw δ observations from both $\bar{r}_L^{I,0}$ and $\bar{k}_L^{I,0}$. For each combination of those draws, the prior mean is equal to the draw from $\bar{r}_L^{I,0}$ and the standard deviation is determined using reverse engineering based on the respective feasible interval for each δ, as prescribed by $\bar{r}_L^{I,0}$ and $\bar{k}_L^{I,0}$. The simulated prior distribution thus has the same mean as $\bar{r}_L^{I,0}$, but a different standard deviation, with $\underline{ROE}_L^{I,0} \sim N(0.0607, 0.0055)$.[51] In this example, the likelihood $\widetilde{ROE}_L^{I,0} \sim N(0.1181, 0.0165)$ has a much larger standard deviation than the overall prior, so that the posterior anchors on the prior. The posterior resulting from the updating process for each δ is equal to $\overline{ROE}_L^{I,0} \sim N(0.0662, 0.058)$, so it has a larger variance than the prior. In cases where prior and likelihood strongly deviate, it is possible that the posterior variance becomes larger than the prior variance; uncertainty

Figure 4

Prior, Likelihood and Posterior for the Long-Term Return on Equity ($\widetilde{ROE}_L^{I,0}$)



[51] The mean and standard deviation are estimated from random draws taken from the δ simulated prior distributions.

increases because the data (through the likelihood) contradicts the prior. As is visible in Figure 4, the 89% CI for $\overline{\mathrm{ROE}}_{\mathrm{L}}^{\mathrm{I},0}$ is [0.0571, 0.0756], meaning that 89% of the posterior distribution are within that interval.

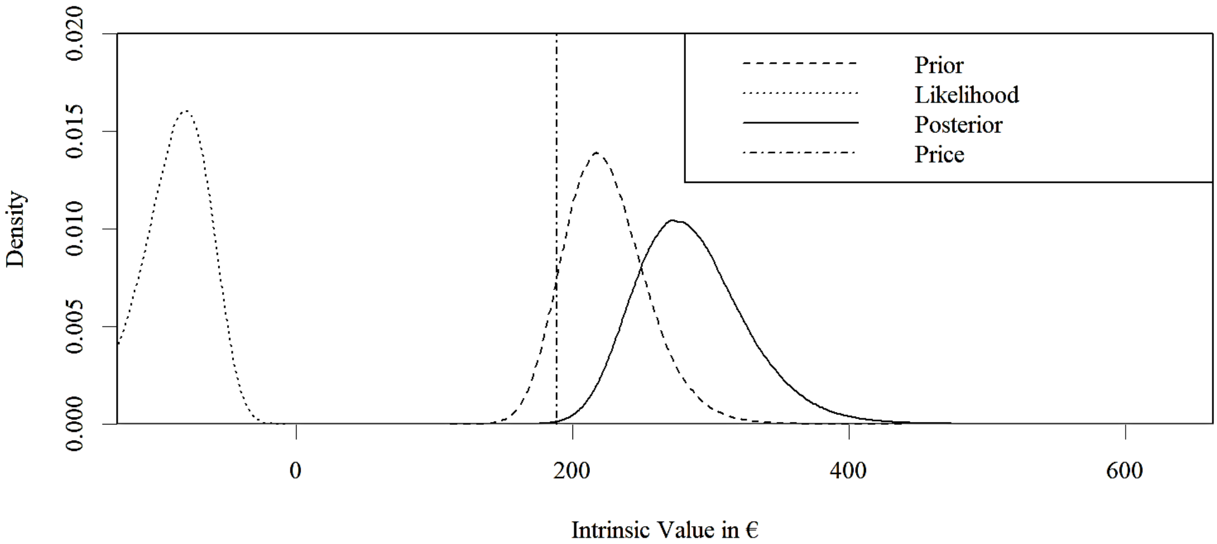
5.4. Simulation of Intrinsic Value and Investment Decision

Finally, the combinations of draws from the three distributions can then be used to simulate $\widetilde{\mathrm{V}}^{\mathrm{I},0}$. That is done by inserting the δ combinations of draws from the prior, likelihood and posterior distributions into Equation (6).[52] The result is a posterior distribution $\overline{\mathrm{V}}^{\mathrm{I},0}$ that can be used for decision-making. The simplest way would be to empirically determine $\mathrm{P}\left[\widetilde{\mathrm{V}}^{\mathrm{I},0} \geq \mathrm{P}_0 | \mathrm{D}, \mathrm{M}\right]$ from the posterior and then set a personal probability threshold, where M is the model that is in turn conditional on its inputs through the prior and likelihood. This probability threshold can be seen as a formal representation of the MoS. That way, investors can answer the main question of interest: what is the probability of $\widetilde{\mathrm{V}}^{\mathrm{I},0}$ to exceed or fall short of P_0, given the model? If that probability exceeds their MoS, then the investment would be undertaken. Without a closed-form solution for $\overline{\mathrm{V}}^{\mathrm{I},0}$, calculating $\mathrm{P}\left[\widetilde{\mathrm{V}}^{\mathrm{I},0} > \mathrm{P}_0 | \mathrm{M}, \mathrm{D}\right]$ through an integral is not possible. In order to approximate $\mathrm{P}\left[\widetilde{\mathrm{V}}^{\mathrm{I},0} > \mathrm{P}_0 | \mathrm{M}, \mathrm{D}\right]$, it is sufficient to simply calculate the share of observations in $\underline{\mathrm{V}}^{\mathrm{I},0}, \widehat{\mathrm{V}}^{\mathrm{I},0}, \overline{\mathrm{V}}^{\mathrm{I},0}$ in the simulated dataset that are larger than P_0.

Figure 5 shows the densities for $\underline{\mathrm{V}}^{\mathrm{I},0}, \widehat{\mathrm{V}}^{\mathrm{I},0}, \overline{\mathrm{V}}^{\mathrm{I},0}$ as well as the market price (displayed as a vertical line) for the example of Volkswagen. Despite the three distributions being non-normal (and skewed), I use the mean and standard deviation of the simulated values for $\underline{\mathrm{V}}^{\mathrm{I},0}, \widehat{\mathrm{V}}^{\mathrm{I},0}$ and $\overline{\mathrm{V}}^{\mathrm{I},0}$ for the sake of comparison. The prior mean $\mu_{\underline{\mathrm{V}}^{\mathrm{I},0}}$ is 224.21€ and the prior standard deviation $\sigma_{\underline{\mathrm{V}}^{\mathrm{I},0}}$ is 31.33, while the likelihood mean $\mu_{\widehat{\mathrm{V}}^{\mathrm{I},0}}$ is -89.82€ and the standard deviation $\sigma_{\widehat{\mathrm{V}}^{\mathrm{I},0}}$ is 28.32.

Figure 5

Prior, Likelihood and Posterior for Intrinsic Value ($\widetilde{\mathrm{V}}^{\mathrm{I},0}$)



Interestingly, all of the simulated values for $\widehat{\mathrm{V}}^{\mathrm{I},0}$ are negative. Simply extrapolating the high short-term ROEs into the long-term future thus does not yield feasible results for Volkswagen. That is because doing so frequently

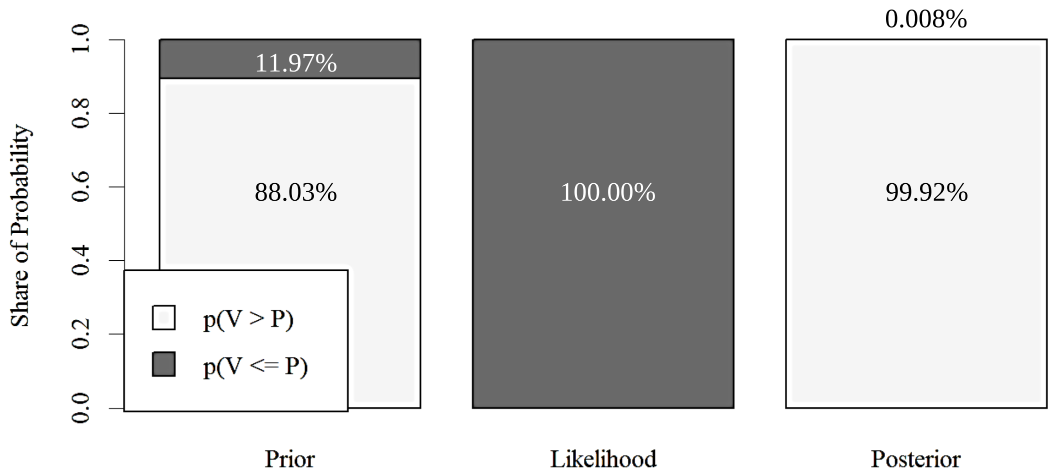
[52] For the mean and standard deviation of $\underline{\mathrm{V}}^{\mathrm{I},0}$, the prior distribution of intrinsic value, the inputs are the δ draws from $\bar{\mathrm{r}}_{\mathrm{L}}^{\mathrm{I},0}$ and $\bar{\mathrm{k}}_{\mathrm{L}}^{\mathrm{I},0}$ since they must be simulated to obtain $\underline{\mathrm{ROE}}_{\mathrm{L}}^{\mathrm{I},0}$ anyway.

violates transversality due to $\widehat{ROE}_L^{I,0} > \hat{r}_L^{I,0}/(1-\hat{k}_L^{I,0})$. When looking at the posterior $\overline{V}^{I,0}$ in the figure, the benefit of adopting Bayesian inference becomes visible since the prior "regulates" the output. In the example, the prior is a bit more conservative than the posterior. That is a logical consequence of "stronger" fundamentals in $\hat{V}^{I,0}$ compared to $\underline{V}^{I,0}$. More importantly, the simulated values for the posterior are all positive and therefore feasible, with the mean $\mu_{\overline{V}^{I,0}}$ of 286.15€ and the standard deviation $\sigma_{\overline{V}^{I,0}}$ of 40.20.

For the investment decision, I calculated the share of simulated values that exceed the market price to proxy for $P[\tilde{V}^{I,0} > P_0|M]$. For the prior, $P[\underline{V}^{I,0} > P_0|M]$ = 88.03%. Without knowing any data aside from the book value and the market price, the investor would be around 88% certain that intrinsic value exceeds price. While that seems quite high, it is due to the prior being centred around book value (since $\mu_{\underline{ROE}_L^{I,0}} \approx \mu_{\underline{r}_L^{I,0}}$), and book value for Volkswagen exceeds the market price by roughly 1 standard deviation of the prior. In this case, $P[\hat{V}^{I,0} > P_0|M,D]$ = 0.00%, since all values of the likelihood are negative.[53] In isolation, the likelihood does not add value to the analysis; it only does when combined with the weakly informative prior. In the posterior, the strong fundamentals relative to the market price further reinforce the impression of the prior, so that $P[\overline{V}^{I,0} > P_0|M,D]$ = 99.92%. Figure 6 plots the probabilities of $\underline{V}^{I,0}$, $\hat{V}^{I,0}$ and $\overline{V}^{I,0}$ to exceed (or fall short of) the market price.

Figure 6

Probabilities of $\underline{V}^{I,0}$, $\hat{V}^{I,0}$ and $\overline{V}^{I,0}$ relative to the Market Price



Based on the posterior, most investors would see their personal MoS exceeded so the investment would be undertaken. It is, however, important to acknowledge that the valuation depends on crucial assumptions in both prior and likelihood and the sensitivity of the results to alternative assumptions should always be tested. Critical assumptions and limitations of the approach that can be tested in future research and valuation practice are:

1) *Normality of Prior and Likelihood*: normality of the expectational values might not be feasible due to autocorrelation of payout ratios and ROEs, especially given that the persistence of these measures is documented in Nissim & Penman (2001) and Huefner & Rueenaufer (2021).

[53] Note that this probability should not be understood as exactly 100% (Cromwell's rule applies). With the number of draws approaching infinity, there will eventually be draws outside of this interval.

2) *Market-based discount rate*: calculating the cost of equity capital based on the (Bayesian) CAPM might leave out important risk-factors and may therefore be biased. In that case, the probability distribution of ROE would also be biased since it anchors on the distribution (but it does not have to).
3) *Weakly informative priors*: alternative, more diffuse priors will produce results that anchor on the likelihood and can thus be seen as less “subjective”. This, however, only works well when the likelihood is within the feasible intervals, so that a more informative prior is not required.
4) *Ad-hoc prior mean for payout*: the mean for the prior of the long-run payout ratio is based on previous empirical results, but those results may not be applicable in each individual case.
5) *Unicorns*: weakly informative priors standardise the results by making “extreme” results very unlikely, but there are firms that deviate from the norm. For these firms, the results will still be feasible (in a mathematical sense) but not ultimately reflective of the business reality.

In summary, applying Bayesian methods to equity valuation does not resolve all of the existing issues, but it raises new points for discussion as it asks different questions compared to frequentist methods that can (and should) be subjects of future research).

6. *Concluding Remarks and Implications for Future Research*

In this paper, I describe equity valuation as the result of many subjective probability assessments under uncertainty, where investors can only rely on the information at hand. I advocate for Bayesian thinking in practical equity valuation because it also allows investors to formally incorporate hard-to-quantify, subjective information and provides a more suitable interpretation of probability. The approach I suggest further provides a (partial) solution for one of the most pervasive issues in equity valuation in both research and practice: accounting for the large amount of uncertainty in equity valuation.

By revealing the uncertainty of investment decisions, Bayesian valuation can be used for intuitive practical decision-making; given the information at hand, what is the probability for intrinsic value to be higher (or lower) than price? Expressing intrinsic value through probability distributions is particularly convenient because it automatically involves a clarification of the margin of safety. But instead of specifying an arbitrary additional discount, the margin of safety requires investors to determine a personal probability threshold.

Empirically, the main advantage of the model I present is that it should produce smaller valuation errors compared to existing frequentist models, because the probability of nonsensical results is minimised through the introduction of weakly informative priors. This applies to cases where the fundamentals are poor indicators of firm value in particular. In turn, the usefulness of the model can be assessed empirically in comparison to existing approaches, e.g. by examining the relation between posterior probabilities and future stock returns.

7. *References*

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